Nonlinear gain and carrier temperature dynamics in semiconductor laser media

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The macroscopic behavior of a semiconductor laser medium is described by use of modified rate equations. The model, valid on time scales greater than 10^{-13} s, explicitly treats carrier temperature as a dynamic variable and includes the nonlinear dependence of the gain function on carrier density and temperature. Gain suppression that is due to carrier heating is a natural consequence of the model and gives a qualitative explanation of subpicosecond gain dynamics experiments without introducing gain nonlinearity phenomenologically. We demonstrate the temperature behavior of the laser during transient dynamics near and well above threshold. By including carrier temperature as a dynamic variable we show that the laser response to an external perturbation exhibits a noticeable change in the damped oscillations of the photon density compared with that in models without temperature dynamics. Variation in the evolution of the gain function for different external pulse energies is also demonstrated. © 1998 Optical Society of America
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1. INTRODUCTION

Gain suppression that is due to carrier heating effects has become an object of special attention during the past decade. Interest in this phenomenon is fueled by the importance of gain dynamics on the picosecond-to-femtosecond time scale in semiconductor laser devices. The results of experiments that investigate ultrafast response of a semiconductor laser amplifier indicate substantial gain suppression,[1] which is recognized to be a consequence of carrier heating effects that are due to the external signal. Although different aspects of the influence of carrier heating on the behavior of diode lasers have also been considered,[2–5] the experimental observation of gain suppression underlines the significance of carrier heating effects on a subpicosecond time scale.

The theory of a semiconductor laser based on standard rate equations[6–8] cannot describe gain suppression that is due to carrier heating. This problem and other problems in semiconductor laser theory can be treated through the development of a comprehensive microscopic theory. This approach was used in the past[9–11] and is also currently under development by several groups of researchers.[12–16] The microscopic (many-body) approach is essential for describing the medium's response on time scales shorter than both the time interval required for establishment of a quasi-equilibrium distribution of carriers and the relaxation time of the induced polarization. To describe the response on longer time scales (10^{-13} s and longer) one can use modified rate equations. This paper is devoted to the development of the latter approach.

We consider a semiconductor laser as a multicomponent system in which there are three subsystems that compose a unified dynamic ensemble. These subsystems are identified as the electrons in the conduction band, the holes in the valence band, and the lattice. They are characterized by a hierarchy of relaxation times in which the intrasubsystem relaxation times are shorter than the intersubsystem relaxation times. For time intervals long enough for quasi-equilibrium to be established, this difference in relaxation times means that the temperatures of these subsystems can be different. On time scales greater than 10^{-12} s,[17] the temperatures of the three subsystems are usually taken to be equal. Thermal equilibrium between the two carrier subsystems is established much more rapidly; this equal-temperature assumption is common in the theory of semiconductors. Therefore, on the subpicosecond time scale, the temperature difference between carriers and the lattice becomes important in the dynamic response of the laser, making it necessary to modify the rate equations for a semiconductor laser by taking into account gain nonlinearities including temperature dependence. Hence, we consider carrier temperature a dynamic variable.

This paper is organized as follows: In Section 2 we

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consider modified rate equations; the temperature dependence of the nonlinear gain is discussed in Section 3. Section 4 includes the basic equations of our model. The steady-state regime of operation, linear approximation, and gain suppression that is due to carrier heating are considered in Section 5. Basic properties of the model and carrier temperature behavior in the cw regime as well as the system response to an external optical signal are demonstrated in Section 6. We summarize our results in Section 7.

2. MODIFIED RATE EQUATIONS

We begin with a set of equations describing the time dependence of the slowly varying amplitude of the electromagnetic field E and the carrier density n in the semiconductor laser:

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -\frac{1}{2\tau_c}(1 + i\Delta)E + 1/2(1 + i\alpha)gE, \qquad (1)$$

$$\frac{\mathrm{d}n}{\mathrm{d}t} = J - \left(\frac{\mathrm{d}n}{\mathrm{d}t}\right)_{\mathrm{sp}} - gW, \qquad (2)$$

where g is the gain function, $W \equiv (\eta\eta_{\mathrm{gr}}/2) \times [|E|^2/(8\pi\hbar\omega)]$ is the density of photons in the cavity, η is the medium's refractive index, $\eta_{\mathrm{gr}}(=c/\nu_{\mathrm{gr}})$ is the group-velocity index, τ_c is the photon lifetime in the cavity, ω is the frequency of emitted photons, $\Delta \equiv 2(\omega_c - \omega)\tau_c$, ω_c is the cavity eigenfrequency, α is the linewidth enhancement factor, and J is the effective pumping rate. The carrier density applies to either electrons or holes, as their concentrations are equal within the local electroneutrality approximation. The term $(\mathrm{d}n/\mathrm{d}t)_{\mathrm{sp}}$ describes the spontaneous recombination of carriers. This term is usually replaced by n/τ_{sp}, where τ_{sp} is a spontaneous recombination time. However, the probability of spontaneous emission depends on the frequency (energy) of the emitted photons. A more rigorous expression for the spontaneous recombination term is derived in Appendix A.

Equations (1) and (2) are widely used to describe semiconductor lasers. The functional dependence of g on n is important in the laser dynamics, and it is relatively common to apply a linear approximation to this function, $g = \gamma_n(n - n_t)$, where γ_n represents the differential gain coefficient and n_t is the carrier density at transparency.

The carrier density is defined by the integral expression

$$n(\mu, T) = \int \rho(\varepsilon)f(\varepsilon, \mu, T)\mathrm{d}\varepsilon, \qquad (3)$$

where

$$f(\varepsilon, \mu, T) = \left[1 + \exp\left(\frac{\varepsilon - \mu}{k_B T}\right)\right]^{-1}$$

is the Fermi–Dirac distribution function, k_B is the Boltzmann constant, $\rho(\varepsilon)$ is the density of states in the conduction band, ε is the energy of an individual electron, and μ is the chemical potential. For definiteness, because n is the same for electrons and holes, we consider electrons whenever the word "carrier" is used and take the zero of energy to be at the top of the valence band.

In recent years carrier temperature has come to be regarded as a separate dynamic variable.[18–24] Treatment of the carrier temperature as a dynamic variable requires modification of rate equations (1) and (2) by an additional rate equation for the carrier temperature or, equivalently, for the carrier energy density U. Such an equation is considered in detail in Refs. 19–24 and elsewhere. Below we consider the logic behind the rate equation for energy density.

The energy density of carriers is defined by the integral expression

$$U(\mu, T) = \int \varepsilon\rho(\varepsilon)f(\varepsilon, \mu, T)\mathrm{d}\varepsilon. \qquad (4)$$

We can derive the rate equation for U by considering energy balance. As the top of the valence band is chosen to be the zero level of energy, the energy of the electron subsystem is reduced by $\hbar\omega$ with each induced recombination and by ε with each spontaneous recombination. The injection current (pumping) supplies the carrier subsystem with particles that have an average energy corresponding to the lattice temperature or higher. Free-carrier absorption of a photon adds energy $\hbar\omega$ to the carrier subsystem. The interaction between the carriers and the lattice leads to thermal equilibrium at the lattice temperature T_0, which is assumed to remain constant because the thermal capacity of the lattice is much greater than that of the carriers. The interaction between electron and hole subsystems leads to thermal equilibrium between them. Thermodynamic equilibrium is established faster between the electron and the hole subsystems than between either of the carrier subsystems and the lattice; therefore the temperatures of the electrons and the holes can be considered equal.

By combining the processes discussed above we arrive at an energy balance equation as follows:

$$\frac{\mathrm{d}}{\mathrm{d}t}U(\mu, T) = -\left(\frac{\mathrm{d}U}{\mathrm{d}t}\right)_{\mathrm{Lat}} - \left(\frac{\mathrm{d}U}{\mathrm{d}t}\right)_{\mathrm{sp}} + \left(\frac{\mathrm{d}U}{\mathrm{d}t}\right)_{\mathrm{pump}} - \hbar\omega gW + \hbar\omega\sigma_f nW. \qquad (5)$$

Here the last two terms are associated with induced interband transitions and free-carrier absorption, respectively, and $\sigma_f = s_f\nu_{\mathrm{gr}}$, where s_f is the free-carrier absorption cross section. The interaction between carriers and lattice is presented in the form of an ordinary relaxation term:

$$\left(\frac{\mathrm{d}U}{\mathrm{d}t}\right)_{\mathrm{Lat}} = \frac{1}{\tau_L}\{U[\mu(T), T] - U[\mu(T_0), T_0]\}, \qquad (6)$$

where τ_L is the carrier–lattice relaxation time. The term for spontaneous recombination is

$$\left(\frac{\mathrm{d}U}{\mathrm{d}t}\right)_{\mathrm{sp}} = \int \frac{1}{\tau_{s\varepsilon}}\varepsilon\rho(\varepsilon)f(\varepsilon, \mu, T)\mathrm{d}\varepsilon, \qquad (7)$$

where $\tau_{s\varepsilon}$ denotes the spontaneous recombination time of carriers with energy ε. In Appendix A we have derived an analytical expression for this term. The pumping term is controlled by the injection current and the tem-

perature of the injected carriers. In homostructure lasers the temperature of injected carriers can be taken equal to the lattice temperature. In heterostructure lasers the average energy of the injected carriers is higher than that of carriers in the active region because of the energy difference between the barrier and the active region; hence there is injection heating.[24]

By combining Eqs. (1), (2), and (5) we can study the laser dynamics both analytically and numerically. For analytical studies it is necessary to obtain functional relationships among the dynamical variables. In the general case these relationships cannot be expressed in closed form. However, with reasonable assumptions analytical expressions can be obtained in some cases, for example, for heavily doped semiconductors; these materials are used in most semiconductor devices.

In heavily doped semiconductors the boundary of the conduction band is vague, so $\rho(\varepsilon)$ is not equal to zero at the bottom of the band and penetrates into the bandgap ($\varepsilon < \varepsilon_g$) in the form of an exponential tail[25–27]:

$$\rho(\varepsilon) = \rho_0 \exp(\varepsilon/\varepsilon_d), \tag{8}$$

where ε is the carrier energy measured from the top of the valence band, ε_d is a band tailing parameter determined by the degree of doping [typical values are approximately 10–30 meV (Refs. 28 and 29)], $\rho_0 \exp(\varepsilon_g/\varepsilon_d)$ is the density of states at the bottom of the conduction band, and ε_g is the band-gap energy of the undoped material.

To simplify the analytical calculations we use the following approximation of the Fermi distribution function:

$$f(\varepsilon, \mu, T) = \begin{cases} 1 - \dfrac{1}{2}\exp\left(\dfrac{\varepsilon - \mu}{k_B T}\right) & \varepsilon \leqslant \mu \\ \dfrac{1}{2}\exp\left(\dfrac{\mu - \varepsilon}{k_B T}\right) & \varepsilon > \mu \end{cases}. \tag{9}$$

The lower the temperature, the closer this approximation is to being the true Fermi function. Figure 1 demonstrates the range of accuracy of the chosen approximation.

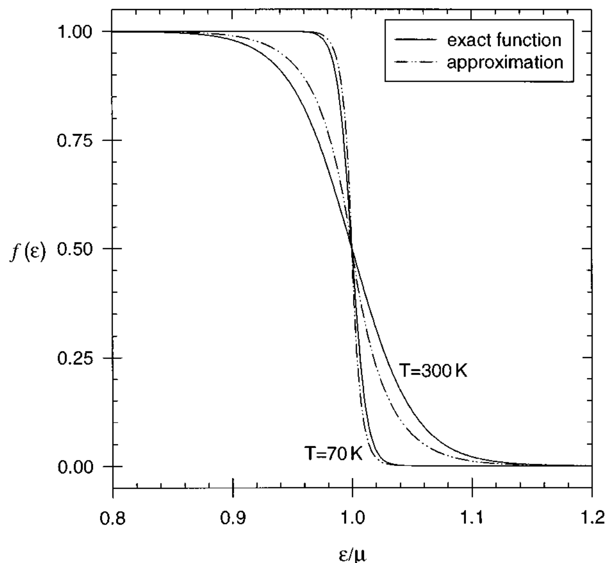

Fig. 1. Exact and approximate Fermi functions at 70 and 300 K.

Calculations of integral (3) and (4), with density function (8) and by use of approximation (9), lead to the following equations:

$$n(\mu, \theta) = \frac{\rho_0 \varepsilon_d}{1 - \theta^2} \exp\left(\frac{\mu}{\varepsilon_d}\right), \tag{10}$$

$$U(\mu, \theta) = \frac{n}{1 - \theta^2}[\mu(1 - \theta^2) + \varepsilon_d(3\theta^2 - 1)], \tag{11}$$

where $\theta = k_B T/\varepsilon_d$. Strictly speaking, these equations are valid only for $\theta < 1$; however, the functional relationship between n and μ goes beyond this limitation (see the discussion in Ref. 28). For $\theta \ll 1$ one can expand Eqs. (10) and (11) in terms of θ and keep only the lowest-order terms:

$$n(\mu, \theta) = \rho_0 \varepsilon_d(1 + \theta^2)\exp(\mu/\varepsilon_d), \tag{12}$$

$$U(\mu, \theta) = n(\mu - \varepsilon_d + 2\varepsilon_d \theta^2). \tag{13}$$

The band tailing parameter ε_d can reach values up to several tens of millielectron volts.[28] Experimental values of ε_d up to 36 meV in a semiconductor laser are reported in Ref. 30, which corresponds to a temperature $T \sim 420$ K. Even at room temperature θ can be less than 1. However, for larger values of θ it is necessary to incorporate a more accurate approximation for $\rho(\varepsilon)$. In doped semiconductors for $\varepsilon \geqslant \varepsilon_g$, $\rho(\varepsilon)$ must smoothly transform from the exponential form into a square-root dependence on $(\varepsilon - \varepsilon_g)$.

The pumping term in Eq. (5) has the meaning of an effective energy flow into the active region owing to carrier injection. For simplicity we use the notation

$$\left(\frac{\mathrm{d}U}{\mathrm{d}t}\right)_{\text{pump}} \equiv Q. \tag{14}$$

In general, Q is an independent parameter of the problem. In homostructure lasers the pumped carriers have the same temperature as the lattice, so we can express Q explicitly, using Eq. (13), as

$$Q = J(\mu_0 - \varepsilon_d + 2\varepsilon_d {\theta_0}^2), \tag{15}$$

where $\theta_0 \equiv k_B T_0/\varepsilon_d$ and μ_0 is determined by the equation

$$J\tau_{\text{sp}} = \rho_0 \varepsilon_d(1 + {\theta_0}^2)\exp(\mu_0/\varepsilon_d). \tag{16}$$

More generally, the phenomenological term that describes the pumping current must be replaced by a term that depends on the gradient of the chemical potential at the p-n junction. This is beyond the scope of this paper and will be considered in future research.

3. GAIN FUNCTION

For further analysis the gain function needs to be specified. A simple form of the gain function was previously derived to include the temperature dependence[20]:

$$g(n, T) = \Gamma(n - n_t - n\beta\Delta T/T_0). \tag{17}$$

Here T_0 is the lattice temperature and ΔT is the deviation of carrier temperature from that of the lattice. By solving Eqs. (1), (2), and (5) along with Eq. (17), we can

demonstrate gain suppression that is due to carrier heating. However, Eq. (17) leaves the values of Γ and β undetermined, and they must be inferred from a quantitative fit to experimental results. Moreover, there are experimental results[1] in which this function fails to give even a qualitative fit (see the discussion in Section 5 below).

A crucial point for further development of the theory is a consistent derivation of the gain coefficient $g(n, T)$. It is highly desirable to obtain an expression that has a form simple enough to permit analytical and numerical analysis of the dynamics of the system yet more comprehensive than Eq. (17). Such an analytical expression can be obtained from the single-particle model of interband transitions, which is a valid approximation on time scales greater than 10^{-13} s. The corresponding expression is given by[28]

$$g(T, \mu, \omega) = G(\omega)\tanh\left(\frac{\mu - \hbar\omega}{2k_B T}\right). \tag{18}$$

The frequency-dependent function $G(\omega)$ has the form

$$G(\omega) = \frac{c^3}{\tau_{\text{sp}}\omega^2\eta^3}\frac{\sqrt{2m_{\text{eff}}{}^3}}{\hbar^2}\sqrt{\hbar\omega - \varepsilon_g} \tag{19a}$$

for transitions from within the conduction band and

$$G(\omega) = \frac{\hbar\pi^2 c^3}{\tau_{\text{sp}}\omega^2\eta^3}\frac{\xi}{\varepsilon_d}\exp\left(\frac{\hbar\omega - \varepsilon_g}{\varepsilon_d}\right) \tag{19b}$$

for transitions from tail states. Here c is the speed of light in free space and ξ is the density of dopants.

4. MODEL EQUATIONS

Having established the conditions for the system of modified rate equations in our model, we can now rewrite Eqs. (1), (2), and (5) in the form

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -\frac{1}{2\tau_c}(1 + i\Delta)E + 1/2[1 + i\alpha]gE + \kappa F, \tag{20a}$$

$$\frac{\mathrm{d}n}{\mathrm{d}t} = J - \left(\frac{\mathrm{d}n}{\mathrm{d}t}\right)_{\text{sp}} - gW, \tag{20b}$$

$$\frac{\mathrm{d}U}{\mathrm{d}t} = -\frac{1}{\tau_L}\{U[\mu(T), T] - U[\mu(T_0), T_0]\} - \hbar\omega gW + \hbar\omega\sigma_f nW + Q - \left(\frac{\mathrm{d}U}{\mathrm{d}t}\right)_{\text{sp}}, \tag{20c}$$

where the spontaneous recombination terms remain in symbolic form. Explicit expressions for these terms are given in Appendix A. The parameters included in these equations can be determined experimentally or calculated on the basis of microscopic theory.

The gain function $g(T, \mu, \omega)$ is determined by Eq. (18). The dynamic variables n and U are connected with μ and T through Eqs. (3) and (4). Because these equations are invertible, μ and T can also be considered dynamic variables. Any pair of functions taken from the different sets $[n, \mu]$ and $[U, T]$ can be chosen as dynamic variables for clarity. In addition, Eqs. (20) are generalized to include the case of a resonant external electromagnetic field $\mathrm{F}(t) = F(t)\exp(-i\omega t)$, where $F(t)$ is a slowly varying function compared with $\exp(-i\omega t)$ and the parameter κ couples the cavity field with the external field. Note that, when the external field is pulsed, Eq. (20a) is valid for pulse durations longer than the cavity photon round-trip time τ_r. If an external pulse has a duration shorter than τ_r, we must use the traveling-wave equation with the corresponding boundary conditions, as was done, for example, in Ref. 31.

To analyze the temperature dynamics we use the following relationship: $\mathrm{d}U/\mathrm{d}t = \varepsilon_n \mathrm{d}n/\mathrm{d}t + \varepsilon_\theta \mathrm{d}\theta/\mathrm{d}t$, where $\varepsilon_n \equiv \partial U/\partial n$ and $\varepsilon_\theta \equiv \partial U/\partial\theta$. By substituting $\mathrm{d}n/\mathrm{d}t$ from Eq. (20b) we can rewrite Eq. (20c) in the following way:

$$\varepsilon_\theta\frac{\mathrm{d}\theta}{\mathrm{d}t} = -\frac{1}{\tau_L}\{U(\mu, T) - U[\mu(T_0), T_0]\} + (\varepsilon_n - \hbar\omega)gW + \hbar\omega\sigma_f nW - \left(\frac{\mathrm{d}U}{\mathrm{d}t}\right)_{\text{sp}} + Q + \varepsilon_n\left(\frac{\mathrm{d}n}{\mathrm{d}t}\right)_{\text{sp}} - J\varepsilon_n. \tag{21}$$

Equation (21) describes either heating or cooling of the subsystem of carriers. Induced interband transitions, in principle, can heat or cool carriers in accordance with the relationship between $\hbar\omega$ and ε_n. Recombination of carriers always decreases the overall energy; however, if recombination takes away less than the average carrier energy, then the subsystem is heated. In the opposite case, cooling takes place.

The term $(\varepsilon_n - \hbar\omega)gW$ in Eq. (21) describes the temperature variation that is due to induced transitions and plays an important role in the nonlinear dynamics of a semiconductor laser. The cw regime of lasing can be unstable if this term corresponds to cooling.[20] In the case of amplification ($g > 0$) the sign of this term is the same as the sign of $\varepsilon_n - \hbar\omega$. Using Eqs. (12) and (13) for n and U, we obtain

$$\varepsilon_n - \hbar\omega = \mu + 2\varepsilon_d\theta_0{}^2 - \hbar\omega. \tag{22}$$

In the case of amplification, $\mu - \hbar\omega > 0$; therefore $\varepsilon_n - \hbar\omega > 0$, and there is no cooling that is due to induced transitions. Hence the temperature instability owing to carrier cooling by induced transitions, which was predicted earlier,[20] does not exist in this model.

5. CARRIER HEATING INFLUENCE ON GAIN

We can obtain the steady-state values of the dynamic variables by setting the left-hand sides of Eqs. (20) to zero. In steady state the gain is equal to the cavity loss; therefore

$$\tanh\left(\frac{\mu - \hbar\omega}{2\varepsilon_d\theta}\right) = \frac{1}{G\tau_c}, \tag{23}$$

which can be solved for μ:

$$\mu = \hbar\omega + \varepsilon_d\theta \ln\left(\frac{G\tau_c + 1}{G\tau_c - 1}\right). \tag{24}$$

For $G\tau_c \gg 1$, which is usually the case (see the estimations below),

$$\mu = \hbar\omega + 2\varepsilon_d\theta/G\tau_c . \tag{25}$$

Consequently, in the steady-state regime μ is close to the energy of the emitted photons, and $(\mu - \hbar\omega)/(2\varepsilon_d\theta) \ll 1$. Using the Taylor expansion, we can now reduce the gain function, Eq. (18), to the following form:

$$g(\theta, \mu, \omega) = G(\omega)\,\frac{\mu - \hbar\omega}{2\varepsilon_d\theta}. \tag{26}$$

Similarly, we can rewrite Eq. (12) in the form

$$n(\mu, \theta) = \rho_0\varepsilon_d(1 + \theta^2)\exp\left(\frac{\hbar\omega}{\varepsilon_d}\right)\exp\left(\frac{\mu - \hbar\omega}{\varepsilon_d}\right)$$
$$\cong n_t\left(1 + \frac{\mu - \hbar\omega}{\varepsilon_d}\right), \tag{27}$$

where the transparency density is defined by

$$n_t \equiv \rho_0\varepsilon_d(1 + \theta^2)\exp(\hbar\omega/\varepsilon_d). \tag{28}$$

Using Eqs. (26) and (27), we can write the gain function as

$$g(n, \theta) = \frac{1}{2}\frac{G}{\theta}\frac{n - n_t}{n_t} = \gamma_n(n - n_t), \tag{29}$$

where the parameter $\gamma_n = G/2n_t\theta$ is the differential gain coefficient. Near room temperature, $\gamma_n \approx 10^{-4}\ \mathrm{cm}^3\ \mathrm{s}^{-1}$, n_t is close to $10^{18}\ \mathrm{cm}^{-3}$, and for $\theta \sim 0.5$ we estimate that $G \approx 10^{14}\ \mathrm{s}^{-1}$.

Equation (29) shows that heating of the carriers leads to gain suppression. For small variations of θ from θ_0 and taking into account the temperature dependence of both γ_n and n_t, we can expand g to first order in $\Delta\theta/\theta_0 \equiv (\theta - \theta_0)/\theta_0$ and obtain

$$g(n, \theta) = \gamma_{n0}[(n - n_{t0})(1 - \Delta\theta/\theta_0) - bn\Delta\theta/\theta_0], \tag{30}$$

with $\gamma_{n0} \equiv \gamma_n(\theta_0)$, $n_{t0} \equiv n_t(\theta_0)$, and $b \equiv 2\theta_0{}^2/(1 + \theta_0{}^2)$. One can see that the gain function [Eq. (30)] is different from that of the linear model given by Eq. (17). If we try to rewrite the gain function [Eq. (30)] in the form of Eq. (17), we find that $\beta = b + 1$, $\Gamma = \gamma_{n0}$, and there exists a second temperature-dependent term, $\gamma_{n0}n_{t0}\Delta\theta/\theta_0$.

It is also relevant to compare Eq. (30) with the following form of the nonlinear gain function, which is common in the literature[19,32,33]:

$$g = \gamma_n(n - n_t)(1 - SW). \tag{31}$$

We can make this comparison by rewriting Eq. (30) in the following way:

$$g = \gamma_{n0}(n - n_{t0})[1 - (b + 1)\Delta\theta/\theta_0]$$
$$- \gamma_{n0}n_{t0}b\Delta\theta/\theta_0, \tag{32}$$

and applying a Taylor expansion for the fractional temperature difference $\Delta\theta/\theta_0$ in terms of W to obtain $\Delta\theta/\theta_0 = a_0 + a_1W + \ldots$. In this expression a_0 and a_1 are parameters that can be calculated from Eq. (21). The parameter a_0 is the fractional carrier temperature deviation when $W = 0$, and it depends on the pumping characteristics. Now we can present the gain function in the same way as in Eq. (31) by writing $\gamma_n = \gamma_{n0}[1 - (b + 1)a_0]$ and $S = [(b + 1)a_1]/[1 - (b + 1)a_0]$. Note, however, that Eq. (32) includes an additional term, $(-\gamma_{n0}n_{t0}b\Delta\theta/\theta_0)$, that also makes a contribution to gain suppression that is due to carrier heating but that is independent of $n - n_{t0}$. This term causes a decrease in g with an increase in carrier temperature whether there is gain, transparency, or absorption. We consider the behavior of the gain function when there is carrier heating in the following three cases:

(a) $n > n_{t0}$ (amplifying medium). The carrier heating leads to a drop in the gain because both temperature-dependent terms in the gain function tend to suppress the gain.

(b) $n = n_{t0}$ (transparent medium). The only nonzero term in Eq. (32) is negative because of carrier heating, making the transparent medium absorptive.

(c) $n < n_{t0}$ (absorbing medium). The two temperature-dependent terms play against each other. One term (proportional to $n - n_{t0}$) tends to increase g, and the other term tends to decrease it. When n is close to n_{t0}, then the second term dominates, i.e., $bn_{t0} \gg (b + 1)|n - n_{t0}|$, and the gain decreases (absorption increases). When $n \ll n_{t0}$, then $bn_{t0} \ll (b + 1)|n - n_{t0}|$, and the gain increases (absorption decreases). Moreover, these gain changes that are due to carrier heating relax much faster than the spontaneous recombination time and can be observed only on the subpicosecond time scale.

Thus, in the cases of gain, transparency, and absorption (for $n \sim n_{t0}$), the gain function has a minimum when the carrier temperature is maximum. This is precisely the behavior that was observed by Kesler and Ippen in their experiment with laser amplifiers.[1] In the case of absorption when n remains much less than n_{t0} the gain function has a peak at the point of maximum carrier heating. We illustrate these features and other possibilities in Section 6.

6. NUMERICAL ANALYSIS

In the numerical simulations we analyze the behavior of the temperature and density of carriers in the conduction band (including tail states). We use Eqs. (20b) and (20c) along with the equation for photon density derived in Appendix C:

$$\frac{\mathrm{d}W}{\mathrm{d}t} + \frac{1}{\tau_c}W - g(n)W + \sigma_f nW = KW_{\mathrm{ext}}, \tag{33}$$

where W_{ext} is the density of photons in an external pulse that can be injected into the laser with coupling coefficient K.

We use the following parameter values in the numerical calculations: spontaneous recombination time $\tau_{\mathrm{sp}} = 2 \times 10^{-9}$ s, temperature (or energy) relaxation time $\tau_L = 5 \times 10^{-13}$ s, band tailing parameter $\varepsilon_d = 20$ meV, dopant concentration $\xi = 10^{18}\ \mathrm{cm}^{-3}$, free-carrier absorption cross section $s_f = 5 \times 10^{-18}\ \mathrm{cm}^2$, photon energy $\hbar\omega$

= 0.989 eV, bandgap energy $\varepsilon_g = 0.990$ eV, refractive index $\eta = 3.319$, group-velocity index $\eta_{\rm gr} = 4$, lattice temperature $T_0 = 70$ K, cavity length $L = 200$ μm, internal losses $\alpha_i = 40$ cm^{-1}, and facet reflectivity $R_1 = R_2 \equiv R = 0.32$. Also, in the numerical calculations we use $q = 3$; for an explanation of this value see Appendix A. The photon lifetime in the cavity is calculated from the expression $\tau_c^{-1} = \nu_{\rm gr}[\alpha_i + (2L)^{-1} \ln(1/R_1R_2)]$. We calculated the material characteristics for an $In_{1-x}Ga_xAs_yP_{1-y}$ quaternary alloy with $y = 0.55$ and $x = 0.25$, using the expressions found in Ref. 7. The values of the other parameters are consistent with values reported in the literature.[6–8,25,28] We chose the lattice temperature as the initial condition for the carrier temperature in each numerical experiment.

Figures 2 show the evolution of the three dynamic variables during a typical laser ($W_{\rm ext} = 0$) turn-on transition through relaxation oscillations to cw behavior. In the cw regime of laser operation the carrier temperature is affected, in general, by processes such as stimulated emission, free-carrier absorption, spontaneous recombination, electron–phonon interactions, and carrier pumping. Other minor effects (e.g., two-photon absorption) are disregarded in this analysis. Overall, these processes keep the carrier temperature above the lattice temperature, as shown in Fig. 2(c).

Figures 3(a) and 3(b) demonstrate the carrier temperature behavior for laser turn-on at different pumping rates ($W_{\rm ext} = 0$). As one can see, the temperature difference between the carriers and the lattice in the cw limit becomes noticeable at high pumping rates. However, we find that this temperature difference has a negligible influence on the output power of the laser. In fact, when we compare the calculated output power with that calculated in the model without temperature dynamics we obtain only ~0.1% difference, even when the pumping current is 25 times the threshold value. At lower pumping rates this difference is even smaller.

The carrier temperature behavior in the time interval between initiation of pumping and laser emission (pumping interval) is determined mainly by spontaneous recombination, electron–phonon interactions, and pumping. For a degenerate electron gas (which is the case in diode lasers and amplifiers) spontaneous recombination is always a heating factor.[34] The electron–phonon interactions tend to bring the carrier subsystem into thermal equilibrium with the lattice. The influence of the pumping on carrier temperature depends on the carrier injection method and the structure of the semiconductor device. For simplicity we assume in our numerical analysis that the temperature of the pumped carriers is equal to the lattice temperature; i.e., there is no injection heating. As we see from Figs. 2(c) and 3(a), and 3(b), the carrier temperature in the pumping interval is slightly higher than the lattice temperature. This temperature elevation is due to spontaneous recombination, an effect known as recombination heating.[35]

To examine the response of the laser to perturbations, we apply resonant external optical signals. Figures 4(a) and 4(b) demonstrate the laser response to Gaussian pulses with 100-ps and 10-ps durations (FWHM), respectively. The external signals are normalized in such a way that both pulses carry the same energy, 0.1 pJ, and the coupling coefficient has the value $K = 1.2 \times 10^{12}$ s^{-1}. The laser is pumped at a rate two times the threshold value. The photon densities are plotted, along

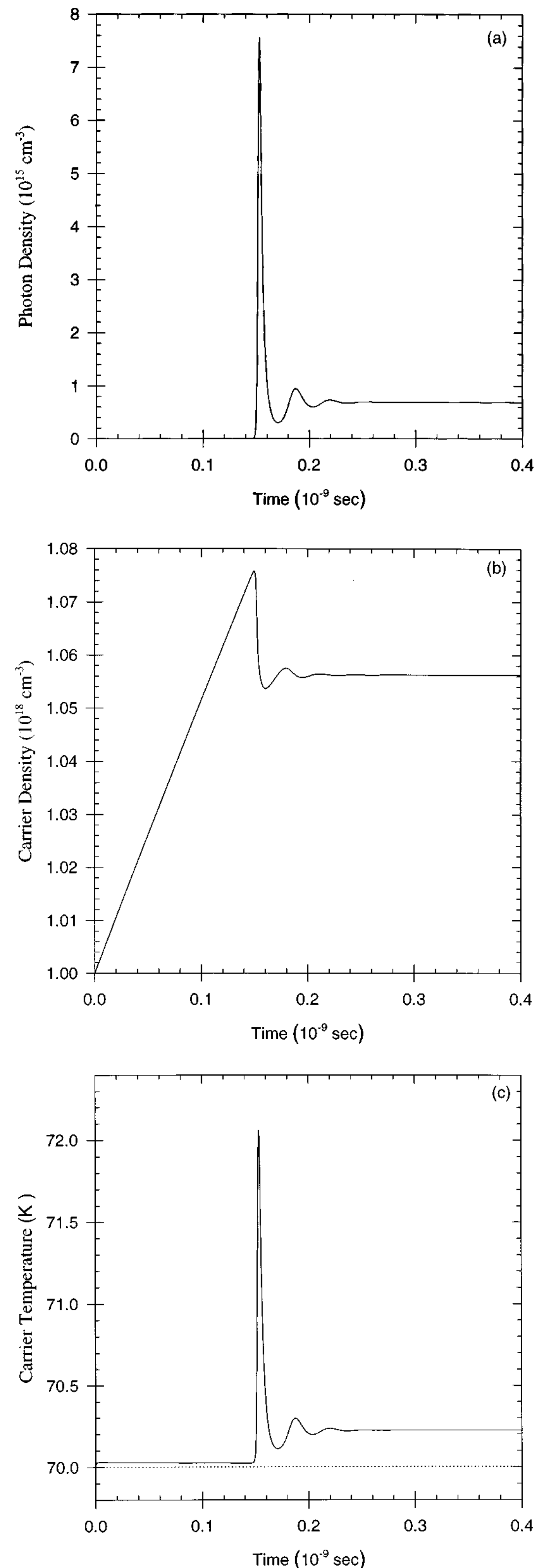

Fig. 2. Laser transition to cw behavior for pumping at twice the threshold rate: (a) photon density, (b) carrier density, (c) carrier temperature (the dotted line represents the lattice temperature).

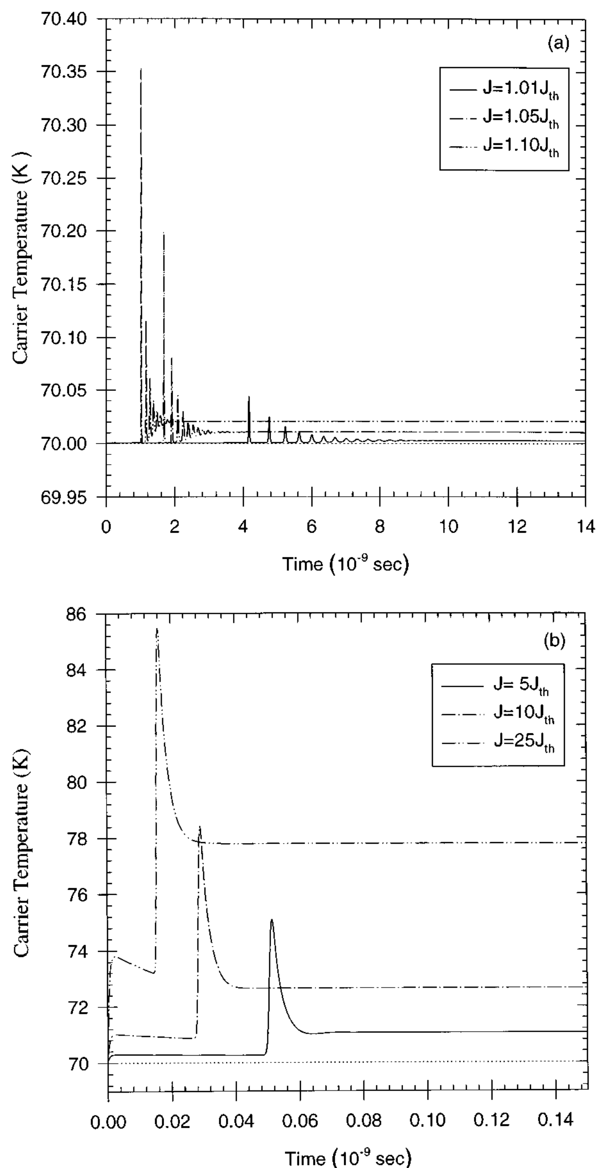

Fig. 3. Carrier temperature behavior (the dotted line represents the lattice temperature) for pumping: (a) slightly above threshold, (b) well above threshold.

with the results of the model assuming equal lattice and carrier temperatures. When the 100-ps pulse is applied, the laser response is virtually the same for the model inclusive of temperature dynamics as for the one without it; however, the response to the 10-ps pulse is different. The oscillations damp faster when the carrier temperature dynamics is taken into account, and this difference becomes more noticeable for shorter signal duration. Thus the carrier temperature dynamics plays a significant role in the dynamic response of the laser on time scales up to several tens of picoseconds.

To demonstrate the gain dynamics of this model on the subpicosecond time scale we apply an external optical signal with a duration of 0.4 ps, a time slightly smaller than the carrier energy relaxation time. For such a short pulse we must either use the traveling-wave equation with corresponding boundary conditions or assume a thin sample of the semiconductor in order to use Eq. (33). We do the latter, using the same parameter values as mentioned above, except for $R = 0.001$ (facets assumed antireflection coated), $L = 10\ \mu\text{m}$ (round-trip time of $\sim$0.3 ps), and $K = 5.2 \times 10^{13}\ \text{s}^{-1}$. We choose the pumping rate in such a way that there is no lasing and keep the energy of the external pulse the same as in the calculations of Fig. 4 (0.1 pJ). We are interested in demonstrating gain dynamics that is due to carrier heating, as discussed in Section 5. Figure 5 demonstrates the gain dynamics (time evolution of the normalized gain function $g\tau_c$) that is due to the external signal, namely, saturation and recovery in the cases of amplification, transparency, and absorption. These results show qualitative agreement between the model developed in this paper and the experimental results of Ref. 1. These figures correspond to the situation when $bn_{t0} \gg (b + 1)|n - n_{t0}|$. Because of this situation we see the dip in the gain function in all three cases.

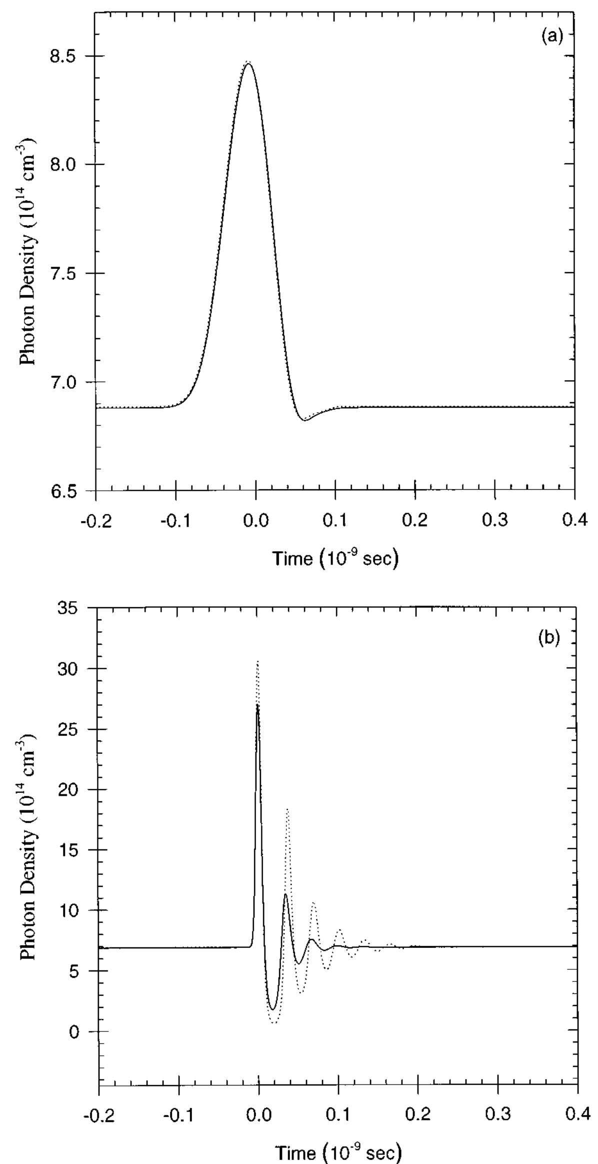

Fig. 4. Laser response (solid curves, with temperature dynamics; dotted curves, without temperature dynamics) to an external 0.1-pJ Gaussian pulse peaked at $t = 0$: (a) 100-ps pulse width, (b) 10-ps pulse width.

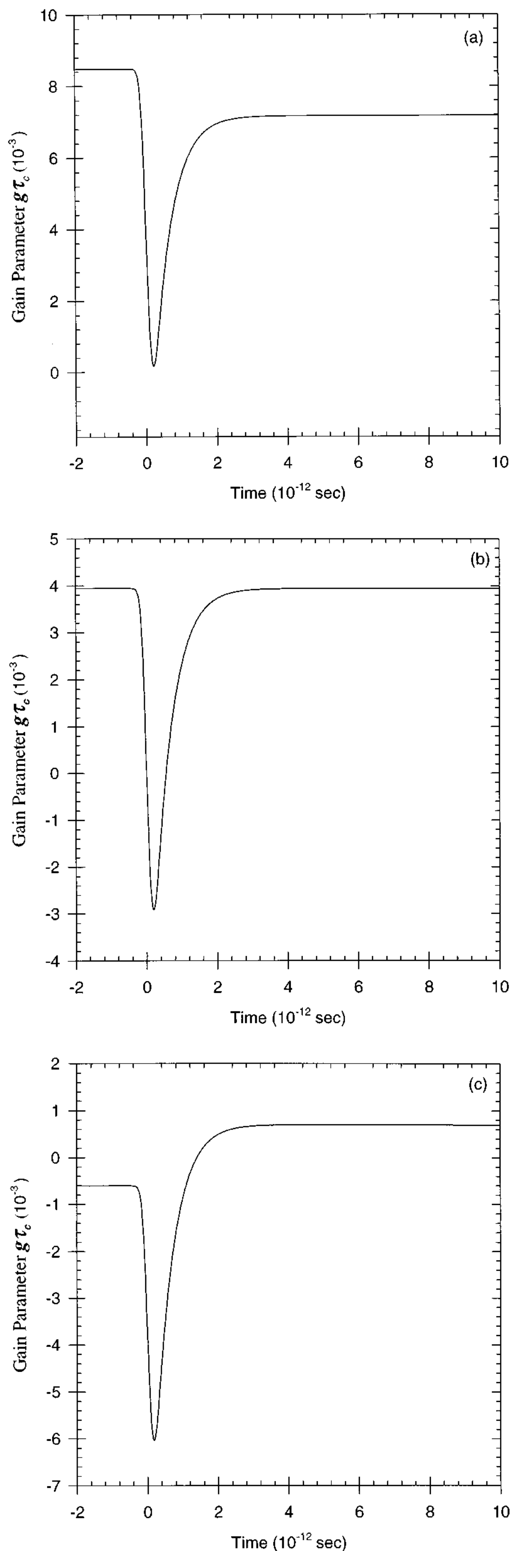

Fig. 5. Subpicosecond gain dynamics; gain saturation owing to carrier heating and recovery in the cases of (a) amplification, (b) transparency, (c) absorption. The 0.1-pJ external pulse is a Gaussian of width 0.4 ps peaked at $t = 0$.

Figure 6 demonstrates the gain dynamics in a strongly absorbing medium that is due to an external optical signal. We apply external 0.4-ps Gaussian pulses with energies ranging from 0.1 to 5.0 pJ. The other parameters are the same as in Fig. 5, except that the pumping rate is chosen to be much smaller, resulting in a much larger absorption coefficient; note the difference in the scale of the gain parameter from that of Fig. 5. As was discussed in Section 5, when the carrier density is sufficiently small the gain function increases because of carrier heating. This behavior is shown in Fig. 6 for the 0.1-pJ pulse. In this case $bn_{t0} \ll (b + 1)|n - n_{t0}|$, and the first temperature-dependent term in Eq. (32) dominates, in contrast with Fig. 5(c) where the second term dominates. This analysis is possible for the 0.1-pJ pulse because our calculations show that the carrier density is not changed enough by the applied pulse to invalidate the inequality. For the higher pulse energies the carrier density changes enough that both temperature-dependent terms of Eq. (32) must be considered in interpreting the behavior of the gain function. For the 0.5- and 1.0-pJ pulses the two carrier heating terms contribute to the gain function with opposite signs. For the 5.0-pJ pulse, when the carrier density becomes close to the transparency density the second term becomes dominant and the increase in the gain function levels off. However, n continues to increase to a value greater than n_{t0}, so the first term again becomes dominant. On the tail of the external pulse the carriers now cool down, further increasing the gain function. As we see, an external pulse can make an absorbing sample transparent and even amplifying for a certain period of time.

In the numerical analysis above we consider gain dynamics that is due to carrier heating only. In reality there are several effects that influence gain dynamics, and they cause differences in the detailed evolution of the gain function. Therefore experimental results can be more complex than those observed by Kesler and Ippen (see, e.g., Refs. 36 and 37).

7. DISCUSSION AND SUMMARY

We have developed a system of modified rate equations describing the interaction of electromagnetic radiation with semiconductors that includes the carrier tempera-

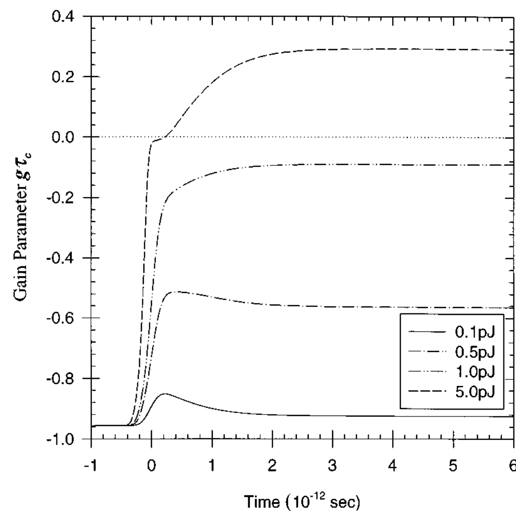

Fig. 6. Subpicosecond gain dynamics in the case of strong absorption for four external pulse energies. In each case the external signal is a 0.4-ps Gaussian pulse peaked at $t = 0$.

ture as a dynamic variable. Our model takes into account the nonlinear functional dependence of the gain coefficient on carrier density and temperature. Expansion of this gain coefficient to lowest order in both density and temperature illustrates that our model includes additional gain suppression that was not taken into account in previous models. This additional gain suppression depends on temperature but not on $[n - n_{t0}]$, and it produces behavior like that observed in subpicosecond gain dynamics experiments in semiconductor laser media.[1]

We investigated the evolution of the dynamical variables during a typical laser turn-on transition to cw behavior. We found that in the cw lasing regime the carrier temperature is always greater than the lattice temperature but that this temperature difference has little effect on the output power. This result is valid for conditions near and also well above threshold.

The temperature dynamics, which is central to our model, is reflected in the behavior of the gain function. We studied this behavior through the response of a laser or a laser medium to an applied optical pulse. Gain dynamics was indirectly observed in the response of a laser to a sufficiently short picosecond pulse, where we showed that there is a noticeable change compared with that of models without the temperature dynamics. However, application of a subpicosecond pulse to a laser medium permits direct observation of gain dynamics. We investigated the response of weakly amplifying, transparent, and weakly absorbing media and in all cases found gain suppression that depends on temperature but not on $[n - n_{t0}]$. By applying pulses of different energies to strongly absorbing media, we saw variation in the evolution of the gain that demonstrates the role played by carrier density in addition to that of carrier heating.

The analytical expressions for carrier density, energy density, and gain function used in our model are mathematically valid for heavily doped semiconductors. However, the region of their validity is wider. Comparison of the gain function given by Eqs. (18) and (19b) with experiment shows excellent agreement between experimental and theoretical curves, even for heterostructure lasers for which the active layer is not necessarily doped.[28] This agreement is a consequence of the fact that, even in undoped semiconductors, the energy dependence of the density of states at the band edge is never described by a square-root function but rather by an exponential (Urbach tails). This is a result of carrier–carrier and carrier–phonon interactions.

One of the purposes of our approach to modeling semiconductor devices was to develop a simple model. In our model analytical relations such as Eqs. (10) and (11) involving carrier density, energy density, and chemical potential simplify both the numerical analysis and the physical interpretation. If we use the general expressions for n and U, Eqs. (3) and (4), the validity of our model is limited only by the duration of the generated or external pulses, which cannot be shorter than the times for the polarization relaxation and the establishment of quasi-equilibrium. However, in this case the numerical analysis becomes more complex and the physical interpretation less transparent.

Treatment of interband recombination of carriers as a many-body effect has become popular in semiconductor laser physics.[12–16] Indeed, for sufficiently large carrier concentration, the interaction between carriers becomes significant. Analysis of the results of the research cited above shows that many-body effects, in general, lead to the so-called Hartree–Fock/Debye correction to carrier energy, which can be considered a carrier-density-dependent renormalization of the bandgap energy. These effects, as well as local field effects, can be included in our model by substitution of the renormalized value of the bandgap energy.

Treatment of semiconductor dynamics on even shorter (femtosecond) time scales requires a truly microscopic approach, such as those described in Refs. 12–16 and 38. The model presented in this paper is valid for any temperature range of practical interest, but greater physical insight is possible at lower temperatures, where the analytical expressions that we have derived are more accurate. Because the system of equations has a three-dimensional phase space, one can expect new, dynamically complex, behavior that should be experimentally verifiable. Additional studies of this behavior are currently under way.

APPENDIX A

As was mentioned in Section 2, the spontaneous recombination term $(\mathrm{d}n/\mathrm{d}t)_{\mathrm{sp}}$ in Eq. (2) is usually replaced by n/τ_{sp}. However, the probability of spontaneous emission depends on the frequency (energy) of emitted photons. Therefore a more rigorous expression for the spontaneous recombination term in Eq. (2) is

$$\left(\frac{\mathrm{d}n}{\mathrm{d}t}\right)_{\mathrm{sp}} = \int_0^\infty \frac{1}{\tau_{s\varepsilon}}\,\rho(\varepsilon) f(\varepsilon,\,\mu,\,T)\mathrm{d}\varepsilon, \tag{A1}$$

where $\tau_{s\varepsilon}$ denote the spontaneous recombination time of carriers with energy ε. We assume that ${\tau_{s\varepsilon}}^{-1} \sim \varepsilon^q$, where $q = 3$ if the matrix element of the dipole moment can be assumed to be independent of frequency. Calculation of the dipole moment in a quantum-well structure gives the frequency dependence ω^{-2} (Ref. 39), implying that $q = 1$. We do not exclude the possibility of other functional dependencies in different cases; therefore, for the sake of generality, we shall keep q as a parameter. Let ${\tau_{s0}}^{-1}$ denote the spontaneous recombination rate of carriers with energy ε_0; then

$$\frac{1}{\tau_{s\varepsilon}} = \frac{1}{\tau_{s0}}\left(\frac{\varepsilon}{\varepsilon_0}\right)^q. \tag{A2}$$

It is convenient to choose $\varepsilon_0 = \varepsilon_g$, where ε_g is the bandgap energy. Now we can use a binomial series and keep only the lower-order terms:

$$\varepsilon^q = [\varepsilon_g + (\varepsilon - \varepsilon_g)]^q = q\varepsilon{\varepsilon_g}^{q-1} - (q-1){\varepsilon_g}^q + \ldots\,. \tag{A3}$$

Such an approximation is reasonable because only energy levels near the bandgap are populated effectively. This is true even at high pumping rates that produce carrier

concentrations of approximately 10^{18}–10^{19} cm^{-3}. Substituting Eqs. (A2) and (A3) into Eq. (A1), we obtain

$$\left(\frac{dn}{dt}\right)_{sp} = \frac{1}{\tau_{s0}}\left[q\,\frac{U(\mu, T)}{\varepsilon_g} - (q-1)n(\mu, T)\right], \quad (A4)$$

where the carrier energy density U is defined in Eq. (4). Similarly, we obtain an expression for the spontaneous recombination term in energy density rate equation (5):

$$\left(\frac{dU}{dt}\right)_{sp} = \frac{1}{\tau_{s0}}[(q+1)U(\mu, T) - q\varepsilon_g n(\mu, T)]. \quad (A5)$$

In the theory of a semiconductor laser the parameter τ_{s0} is assumed to be independent of the carrier density. Sometimes a linear dependence of τ_{s0} on density is used[40]; however, as shown in Appendix B, τ_{s0} has a linear dependence on the carrier density in the case of Boltzmann statistics and is independent of the carrier density in the case of a degenerate ensemble. Therefore the independence of τ_{s0} on carrier density is a good approximation for a semiconductor laser.

In doped semiconductors the bottom of the conduction band is not well defined, and therefore the parameter ε_g is not uniquely determined as in the case of pure semiconductors. Because of this, the calculation for the spontaneous emission term in doped semiconductors requires special consideration.

To calculate the spontaneous recombination terms for heavily doped semiconductors we use the general equations (A1) and (7), with Eqs. (8) and (9). As before, we take Eq. (A2) for the spontaneous recombination rate; however, in this case we use in place of Eq. (A3) the following expansion:

$$\frac{1}{\tau_{s\varepsilon}} = \frac{1}{\tau_{s0}}\left(\frac{\mu}{\varepsilon_0}\right)^q\left(1 + \frac{\varepsilon-\mu}{\mu}\right)^q \cong \frac{1}{\tau_{s0}}\left(\frac{\mu}{\varepsilon_0}\right)^q \times\left[1 + q\,\frac{\varepsilon-\mu}{\mu} + \frac{q(q-1)}{2!}\left(\frac{\varepsilon-\mu}{\mu}\right)^2 + \ldots\right]. \quad (A6)$$

This expansion is used because in heavily doped semiconductors the energy levels near μ are the most heavily populated. To lowest order in θ [defined after Eq. (11)], the recombination terms are

$$\left(\frac{dn}{dt}\right)_{sp} = \frac{n}{\tau_{s0}}\left(\frac{\mu}{\varepsilon_0}\right)^q\left(1 - q\,\frac{\varepsilon_d}{\mu} + 2q\,\frac{\varepsilon_d}{\mu}\,\theta^2\right), \quad (A7)$$

$$\left(\frac{dU}{dt}\right)_{sp} = \frac{n}{\tau_{s0}}\left(\frac{\mu}{\varepsilon_0}\right)^q[\mu - (q+1)\varepsilon_d + 2(q+1)\varepsilon_d\theta^2]. \quad (A8)$$

By definition ε_0 is determined by τ_{s0}. Choosing τ_{s0} to be the experimentally measured spontaneous lifetime of carriers near the bottom of the conduction band of the corresponding undoped semiconductor then gives $\varepsilon_0 = \varepsilon_g$.

APPENDIX B

Three main processes contribute to the spontaneous recombination of carriers[41]: spontaneous radiative recombination, recombination involving traps, and Auger recombination. The rate of recombination involving traps is proportional to the carrier density n over a wide range of n. The Auger recombination rate is proportional to n^3 and for $n \leqslant 10^{19}$ cm^{-3} is significantly less than the spontaneous radiative and trap-involving recombination rates. The functional dependence of the spontaneous radiative recombination rate on the density of carriers depends on the degree of statistical degeneracy of the carriers. We discuss this problem below.

The spontaneous radiative recombination rate can be presented in general form as

$$B(n, T) = \frac{4}{(2\pi\hbar)^6}\int\int w_{cv}(p, p')f_e{}^c(p)f_n{}^v(p')\mathrm{d}p\,\mathrm{d}p', \quad (B1)$$

where $w_{cv}(p, p')$ is the probability of interband recombination of carriers with momenta p and p', and $f_e{}^c$ and $f_h{}^v$ are distribution functions of carriers in the corresponding conduction and valence bands. The factor 4 appears because of the spin degeneracy of electrons and holes.

We can simplify integral (B1) by taking into account the following circumstances: (i) In an isotropic material, distribution functions depend only on energy and (ii) in direct gap semiconductors, optical transitions preserve the momentum of carriers. This means that $w_{cv}(p, p') \cong w(\varepsilon)\delta(p - p')$, and Eq. (B1) can be rewritten in the following form:

$$B(n, T) = \frac{4C}{(2\pi\hbar)^3}\int w(\varepsilon)f_c\left(\frac{m}{m_e}\,\varepsilon\right)f_v\left(\frac{m}{m_h}\,\varepsilon\right)\sqrt{\varepsilon}\mathrm{d}\varepsilon, \quad (B2)$$

where $m_{e,h}$ are electron and hole effective masses in the conduction and the valence bands, respectively, and

$$\varepsilon = \frac{p^2}{2m}, \quad \frac{1}{m} = \frac{1}{m_e} + \frac{1}{m_h},$$

$$C = \frac{4\pi\sqrt{2}m^{3/2}}{(2\pi\hbar)^3},$$

$$f_{c,v}(\varepsilon) = \left[\exp\left(\frac{\varepsilon-\mu_{e,h}}{k_B T}\right) + 1\right]^{-1}. \quad (B3)$$

The probability of transition $w(\varepsilon)$ depends on the carrier energy $\varepsilon_g + \varepsilon$. As far as ε is determined by the carrier temperature, we can take $\varepsilon_g \gg \varepsilon$ and neglect the energy dependence of the probability of recombination. Therefore we can replace $w(\varepsilon)$ by the constant w_0.

The density of the carriers is defined by

$$n_{e,h} = 2C\left(\frac{m_{e,h}}{m}\right)^{3/2}\int_0^\infty\left[\exp\left(\frac{\varepsilon-\mu_{e,h}}{k_B T}\right) + 1\right]^{-1}\sqrt{\varepsilon}\mathrm{d}\varepsilon. \quad (B4)$$

If the ensemble of carriers is far from degeneracy, $\exp[(\varepsilon - \mu_{e,h})/(k_B T)] \gg 1$, and in this case we obtain

$$n_{e,h} = C\sqrt{\pi}\left(\frac{m_{e,h}}{m}\,k_B T\right)^{3/2}\exp\left(\frac{\mu_{e,h}}{k_B T}\right), \quad (B5)$$

$$B(n,\ T) = n_e n_h \int B(T,\ \varepsilon)\mathrm{d}\varepsilon = n_e n_h B(T), \tag{B6}$$

where

$$B(T) = \frac{2}{\sqrt{\pi}} \frac{1}{(2\pi\hbar)^3} \frac{1}{C} \left(\frac{m^2}{m_e m_h}\right)^{3/2} \frac{w_0}{(k_B T)^{3/2}}. \tag{B7}$$

Under the condition of electroneutrality, $n_e = n_h \equiv n$, Eq. (B6) leads to a quadratic dependence of the spontaneous recombination rate on carrier density.

Consider the case of degeneracy when the distribution function is equal to 1 for $\varepsilon \leqslant \mu_{e,h}$ and is equal to 0 for $\varepsilon \geqslant \mu_{e,h}$. Calculation of integrals (B2) and (B4) gives

$$n_{e,h} = \frac{4}{3}\left(\frac{m_{e,h}}{m}\,\mu_{e,h}\right)^{3/2} C,$$

$$B(n,\ T) = 2\frac{w_0}{(2\pi\hbar)^3}\frac{4}{3}\left(\frac{m_{e,h}}{m}\,\mu_{e,h}\right)^{3/2},$$

$$C = 2\frac{w_0}{(2\pi\hbar)^3}\,n. \tag{B8}$$

Thus in this case the spontaneous recombination rate is proportional to the density of carriers; therefore we can introduce the spontaneous recombination time

$$\frac{1}{\tau_{\mathrm{sp}}} = 2\frac{w_0}{(2\pi\hbar)^3}. \tag{B9}$$

Apparently the spontaneous recombination rate cannot be described by a simple analytical formula in any range of densities and temperatures. Therefore we propose an approximation of integral (B6) in the form

$$B(n,\ T) = B(T)\,\frac{n^2}{1 + n/n_0}. \tag{B10}$$

The ratio n/n_0 is known as the degeneracy parameter.[42] At $n \ll n_0$, Eq. (B10) is identical to the case of the Boltzmann statistics of the carriers. Now we require that Eq. (B10) be identical to Eqs. (B8) in the other limiting case, when $n \gg n_0$. From this condition we obtain

$$n_0 = 2\left(\frac{M k_B T}{2\pi\hbar^2}\right)^{3/2}, \quad M = m_e + m_h. \tag{B11}$$

This is an expression for the degeneracy concentration of the Fermi ensemble of particles with mass M.[43] The typical situation in semiconductors is $m_e \ll m_h$, and n_0 is practically equal to the degeneracy concentration of holes and significantly higher than that of the electrons in the conduction band. In a semiconductor laser we deal with a statistically degenerate ensemble of carriers and, therefore, the assumption of linear dependence on carrier density for spontaneous radiative recombination is justified.

APPENDIX C

We begin with the equation describing the time dependence of the slowly varying amplitude of the electromagnetic field E:

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -\frac{1}{2\tau_c}(1 + i\Delta)E + 1/2[1 + i\alpha]gE + \kappa F, \tag{20a}$$

and consider the external signal as a multimode field with a random modal phase distribution:

$$F(t) = \sum_{\Omega} F_{\Omega}(t)\exp[-i(\Delta\Omega t + \phi_{\Omega})], \tag{C1}$$

where $\Delta\Omega = \Omega - \omega$ and φ_{Ω} is randomly distributed in the interval $(0, 2\pi)$.

To derive the equation for photon density we write a formal solution of Eq. (20a) in the form

$$E = \kappa \exp[-(1/2)D(t)]\int_0^t \exp[(1/2)D(t')]F(t')\mathrm{d}t', \tag{C2}$$

where

$$D(t) = \int_0^t \left[\frac{1}{\tau_c} - g + i(\Delta - \alpha g)\right]\mathrm{d}t'. \tag{C3}$$

Solution (C2) satisfies the initial condition that $E = 0$ at $t = 0$. This initial condition is chosen for convenience, because the final result does not depend on initial conditions. In principle we can satisfy any initial condition if we add to Eq. (C2) the general solution of Eq. (20a) for $F = 0$. We calculate the value EE^*, which is proportional to the photon density in a cavity:

$$EE^* = \kappa^2 \exp[-\mathrm{Re}\ D(t)]\int_0^t \mathrm{d}t' \int_0^t \mathrm{d}t'' \times \exp[(1/2)D(t') + (1/2)D^*(t'')]F(t')F^*(t''). \tag{C4}$$

Now we must calculate

$$F(t')F^*(t'') = \sum_{\Omega,\Omega'} F_{\Omega}(t')F_{\Omega'}{}^*(t'') \times \exp[-i(\Delta\Omega t' - \Delta\Omega' t'' + \phi_{\Omega} - \phi_{\Omega'})]. \tag{C5}$$

Assume that the external signal has a broadband spectrum so that $F_{\Omega}(t)$ is a slowly varying function of Ω. Now split the sum [Eq. (C5)] into two parts:

$$\sum_{\Omega,\Omega'} F_{\Omega}(t')F_{\Omega'}{}^*(t'')\exp[-i(\Delta\Omega t' - \Delta\Omega' t'' + \phi_{\Omega} - \phi_{\Omega'})] = \sum_{\Omega} F_{\Omega}(t')F_{\Omega}{}^*(t'')\exp[-i\Delta\Omega(t' - t'')] + \sum_{\Omega\neq\Omega'} F_{\Omega}(t')F_{\Omega'}{}^*(t'') \times \exp[-i(\Delta\Omega t' - \Delta\Omega' t'' + \phi_{\Omega} - \phi_{\Omega'})]. \tag{C6}$$

Inasmuch as $F_{\Omega}(t)$ is a slowly varying function of Ω we can use the following approximation:

$$\sum_{\Omega} F_{\Omega}(t')F_{\Omega}{}^{*}(t'')\exp[-i\Delta\Omega(t'-t'')]$$

$$\approx F_{\omega}(t')F_{\omega}{}^{*}(t'')\sum_{\Omega} \exp[-i\Delta\Omega(t'-t'')], \quad \text{(C7)}$$

$$\sum_{\Omega\neq\Omega'} F_{\Omega}(t')F_{\Omega'}{}^{*}(t'')$$
$$\times \exp[-i(\Delta\Omega' t' - \Delta\Omega t'' + \phi_{\Omega} - \phi_{\Omega'})]$$
$$\approx F_{\omega}(t')F_{\omega}{}^{*}(t'')\sum_{\Omega\neq\Omega'}$$
$$\times \exp[-i(\Delta\Omega' t' - \Delta\Omega t'' + \phi_{\Omega} - \phi_{\Omega'})]$$
$$= 0. \quad \text{(C8)}$$

The last sum is equal to zero because the phases in the exponent are randomly distributed in the interval $(0, 2\pi)$. Now we replace the sum on the right-hand side of relation (C7) by the integral

$$F_{\omega}(t')F_{\omega}{}^{*}(t'')\sum_{\Omega} \exp[-i\Delta\Omega(t'-t'')]$$
$$\approx \frac{F_{\omega}(t')F_{\omega}{}^{*}(t'')}{\Delta\Omega_0}\int \exp[-i\Delta\Omega(t'-t'')]\mathrm{d}(\Delta\Omega)$$
$$\approx 2\pi \frac{F_{\omega}(t')F_{\omega}{}^{*}(t'')}{\Delta\Omega_0}\delta(t'-t''), \quad \text{(C9)}$$

where $\Delta\Omega_0$ is the frequency interval between adjacent modes. After substitution of relation (C9) into Eq. (C4) and integration over t'' we obtain

$$EE^{*} = 2\pi \frac{\kappa^2}{\Delta\Omega_0}\exp[-\mathrm{Re}\, D(t)]$$
$$\times \int_0^t \exp[\mathrm{Re}\, D(t')]|F_{\omega}(t')|^2 \mathrm{d}t'. \quad \text{(C10)}$$

Converting Eq. (C10) back to a differential equation and multiplying by $\eta\eta_{\mathrm{gr}}/(16\pi\hbar\omega)$, we obtain

$$\frac{\mathrm{d}W}{\mathrm{d}t} + \frac{1}{\tau_c}W - g(n)W + \sigma_f n W = KW_{\mathrm{ext}}, \quad \text{(C11)}$$

where $W_{\mathrm{ext}}(t) \equiv [\eta\eta_{\mathrm{gr}}/(\Delta\Omega_0\tau_c 16\pi\hbar\omega)]|F_{\omega}(t)|^2$ is the density of external signal photons, $K \equiv 2\pi\kappa^2\tau_c$, and the term describing the free-carrier absorption (which is small compared with interband absorption) is introduced phenomenologically.

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