

Slow light in an optomechanical microresonator system

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ABSTRACT

Slow light, i.e., the delay of an incident resonant pulse, can be observed in the throughput of an optical whispering-gallery microresonator. It can be produced by a single overcoupled whispering-gallery mode (WGM), or, more usefully, through induced transparency effects that are observed in the case of two coresonant WGMs with very different quality factors. There are several different methods for achieving induced transparency, two of which will be considered here. In addition, under the right conditions, light in a WGM can excite acoustic WGMs by forward Brillouin scattering. This nonlinear process due to electrostriction has a threshold, above which energy is transferred from the first optical WGM to the acoustic WGM and to a lower-frequency optical WGM. When one of the optical WGMs taking part in this optomechanical process is also involved in the production of slow light, the pulse delay can be affected. Analytical expressions for pulse delay in the three cases mentioned above are examined in terms of the WGM intracavity powers and it is shown that when the higher-frequency optical WGM is responsible for slow light, the pulse delay is reduced when the optomechanical process occurs. This conclusion is verified by a numerical model.

Keywords: slow light, microresonator, whispering-gallery modes, optomechanics

1. INTRODUCTION

The throughput of a single microresonator can exhibit induced transparency effects, accompanied by pulse delay (slow light). For example, these can be observed when tunable laser light is injected via a tapered-fiber coupler into a dielectric microresonator such as a fused-silica hollow bottle resonator^{1,2} (HBR) that has two coresonant (frequency-degenerate) whispering-gallery modes (WGMs) having very different quality factors (Q s).³ Two methods involving a pair of copropagating WGMs will be considered here. Coresonance can happen by coincidence, but it can also be introduced in a controllable way by strain tuning (axial stretching of the HBR). Whispering-gallery microresonators have two orthogonally polarized families of modes, TE (transverse electric) and TM (transverse magnetic). Because the birefringence induced by strain tuning causes the two types of modes to tune at different rates, it can be used to impose frequency degeneracy between a TE mode and a TM mode.⁴⁻⁶ Strain tuning also causes WGMs of the same polarization but different radial orders to frequency-shift at different rates, and can be used to make them coresonant.⁷⁻⁹

One method for achieving induced transparency, not considered here, uses cross-polarization coupling between WGMs of orthogonal polarizations. The first method we will consider uses incident light linearly polarized at 45° (for example) in the TE-TM basis to drive coresonant modes of the two polarizations and produce induced transparency in the throughput of the same linear polarization as the incident light. This occurs even in the absence of cross-polarization mode coupling, demonstrating that mode superposition is sufficient to produce these effects. The effect induced in this manner is referred to as coresonant polarization induced transparency (CPIT).^{10,11} An input pulse whose center frequency is resonant will be delayed. These effects are similar to the coupled-resonator-induced transparency (CRIT) observed in coupled whispering-gallery microresonators.^{12,13}

For the second method, linearly polarized light is input and excites only TE (or TM) modes; two modes, of like polarization but different radial order, are coresonant and coupled to each other. In this case, the coupling between modes is mediated by the input/output coupling fiber – light circulating in one WGM couples out into the fiber and then immediately back into the other WGM.^{7,14,15} We use FMIT to refer to the fiber-mediated induced transparency effects seen using this method. Finally, of course, a single WGM can also produce pulse delay, provided that it is overcoupled (coupling loss greater than intrinsic loss).

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Stimulated Brillouin scattering in the microresonator leads to the excitation of acoustic WGMs, consisting of surface acoustic resonances in the frequency range of tens or hundreds of MHz.¹⁶⁻¹⁸ The process is forward Brillouin scattering, which leads to the excitation of a lower-frequency, copropagating, optical WGM. Phase matching of this process is enabled by the different effective refractive indices of optical WGMs of different radial orders. When the input optical power is large enough ($\sim 100 \mu\text{W}$, for optical $Q \sim 10^8$) to reach threshold, the circulating intracavity powers in the optical WGMs are changed. If one of those WGMs is also responsible for slow light, the pulse delay can be affected.

A simple ring-cavity model was developed to study induced transparency and slow light in microresonators. From this model, analytical expressions for pulse delay were derived.¹⁹ These expressions are now shown to depend on intracavity power (actually, intracavity enhancement of the input power). Since excitation of acoustic WGMs affects the intracavity power, it also affects the pulse delay. The model¹⁹ has been extended to include the acoustic WGM and its nonlinear interaction with two optical WGMs. Numerical results from the model confirm the analytically predicted behavior, specifically the reduction in delay when the higher-frequency WGM is involved in producing the slow light.

2. MODEL AND PULSE DELAY

The ring-cavity model outlined in Ref. 19 gave expressions for the pulse delay in the two types of induced transparency (CPIT, FMIT) considered here. In both of these cases, the expressions for (group) delay τ_g are completely general; that is, they do not depend on any assumptions as to input/output coupling regime (overcoupled or undercoupled) or the Q values of the two WGMs. The coupling regime is described in terms of the ratio of coupling loss T_j to intrinsic loss $\alpha_j L$, $x_j = T_j / \alpha_j L$ ($j = 1$ or 2 specifies the mode and L is the resonator circumference); a ratio < 1 denotes undercoupling, a ratio $= 1$ indicates critical coupling, and a ratio > 1 means overcoupling. Strong overcoupling thus implies $x_j \gg 1$. Other terms appearing in the expressions are n , the effective refractive index of the modes, a , the resonator radius, and c , the speed of light. Though the two WGMs can have different refractive indices, the difference is negligible here.

In the case of CPIT with 45° linear polarization, the group delay is given by

$$\tau_g = \left(\frac{4\pi ma}{c} \right) \frac{\frac{x_1^2}{T_1}(x_2+1)^2 + \frac{x_2^2}{T_2}(x_1+1)^2}{(x_1+1)(x_2+1)(x_1x_2-1)}. \quad (1)$$

In the limits of both WGMs being strongly overcoupled, and further $Q_2 \gg Q_1$ (T_j is inversely proportional to Q_j in the strongly overcoupled limit), this becomes

$$\tau_g \xrightarrow{x_j \gg 1} \frac{4\pi ma}{c} \left(\frac{1}{T_1} + \frac{1}{T_2} \right) \xrightarrow{Q_2 \gg Q_1} \frac{4\pi ma}{c} \left(\frac{1}{T_2} \right). \quad (2)$$

For FMIT, the group delay is

$$\tau_g = \left(\frac{8\pi ma}{c} \right) \frac{\frac{x_1^2}{T_1} + \frac{x_2^2}{T_2}}{(x_1 + x_2)^2 - 1}. \quad (3)$$

In the limits of both WGMs strongly overcoupled with equal throughput dip depths so that $x_1 = x_2$, and further $Q_2 \gg Q_1$, this becomes

$$\tau_g \xrightarrow{x_1=x_2 \gg 1} \frac{2\pi ma}{c} \left(\frac{1}{T_1} + \frac{1}{T_2} \right) \xrightarrow{Q_2 \gg Q_1} \frac{2\pi ma}{c} \left(\frac{1}{T_2} \right). \quad (4)$$

By restricting the model to a single optical WGM, it is easy to show that the group delay produced in this case is

$$\tau_g = \left(\frac{4\pi ma}{c} \right) \frac{x^2}{x^2 - 1}. \quad (5)$$

This becomes, in the strongly overcoupled limit,

$$\tau_g \xrightarrow{x \gg 1} \frac{4\pi ma}{c} \left(\frac{1}{T} \right). \quad (6)$$

For the single WGM, the intracavity enhancement can be written as the ratio of intracavity power P_s to input power P_i :

$$\frac{P_s}{P_i} = 4 \frac{x^2}{(x+1)^2} \xrightarrow{x \gg 1} \frac{4}{T}. \quad (7)$$

Thus we see that, at least in the strongly overcoupled limit, the single-mode pulse delay is proportional to the intracavity power (for a given input power). In the case of CPIT, where the two modes are independent, this will also be the case. For FMIT, because the intermode coupling is quite weak, the expression in Eq. (7) is still a good approximation to the intracavity power in each mode. We thus conclude that the pulse delay will be roughly proportional to the intracavity power (actually, intracavity enhancement) in all cases considered here.

3. EFFECTS OF ACOUSTIC EXCITATION

The two optical WGMs giving rise to induced transparency have been labeled modes 1 and 2, with mode 2 having the higher Q . The single WGM will also be labeled mode 2. Now consider the case of mode 2 being coupled to a lower-frequency optical WGM, mode 3, through an acoustic WGM. (It will be assumed that mode 1, coresonant with mode 2, does not couple to mode 3 in this way; if it does, the qualitative results will be the same.) The acoustic WGM frequency is taken in the model to be equal to the frequency difference between modes 2 and 3, and for phase matching the acoustic wave number is equal to the difference in optical wave numbers. For this to obtain for forward Brillouin scattering, the following relation must hold:

$$\Delta n = \frac{c}{v} \left(\frac{\omega_2 - \omega_3}{\omega_3} \right), \quad (8)$$

where $\Delta n = n_2 - n_3$ is the difference in effective refractive index of the two modes, c is the speed of light, v is the acoustic speed, and ω_2 and ω_3 are the mode frequencies. This index difference, on the order of 0.02, can be achieved by using optical WGMs of different radial orders.

Figure 1 shows the throughput frequency spectra for the three cases considered here – CPIT, FMIT, and single WGM. The higher-frequency modes (or mode) are responsible for the pulse delay when the center frequency of the input pulse is resonant with mode 2 (and mode 1, for CPIT and FMIT). In Fig. 1, the spectra are shown for low input power because, above threshold, beats will appear on the higher-frequency mode due to the simultaneous excitation of mode 2 directly and mode 3 via Brillouin scattering. For purposes of illustration, the 2-3 frequency difference is taken to be 50 MHz, equal to the acoustic WGM frequency, and $Q_2 = Q_3 = 10^8$ while $Q_1 = 10^7$. All modes are strongly overcoupled, with all $x = 78$ for CPIT and all $x = 5.8$ for FMIT and single WGM. The upper trace, showing the resonance dips, is the throughput power. The peaks shown below the throughput trace are the intracavity powers (attenuated by 10^5): the green peak at lower frequency represents mode 3, and the red peak at higher frequency is mode 2. Induced transparency is evident in the first two cases.

Because of the beating above threshold, it can be difficult to evaluate the effect of acoustic excitation on the pulse delay. Therefore, a modified pulse response was calculated and is displayed in Fig. 2. The pulsed input field was amplitude modulated (100% depth of modulation) at 25 MHz, so that its frequency spectrum consists only of two sidebands separated by 50 MHz. Its center frequency was then tuned to the midpoint between modes 2 (& 1) and 3.

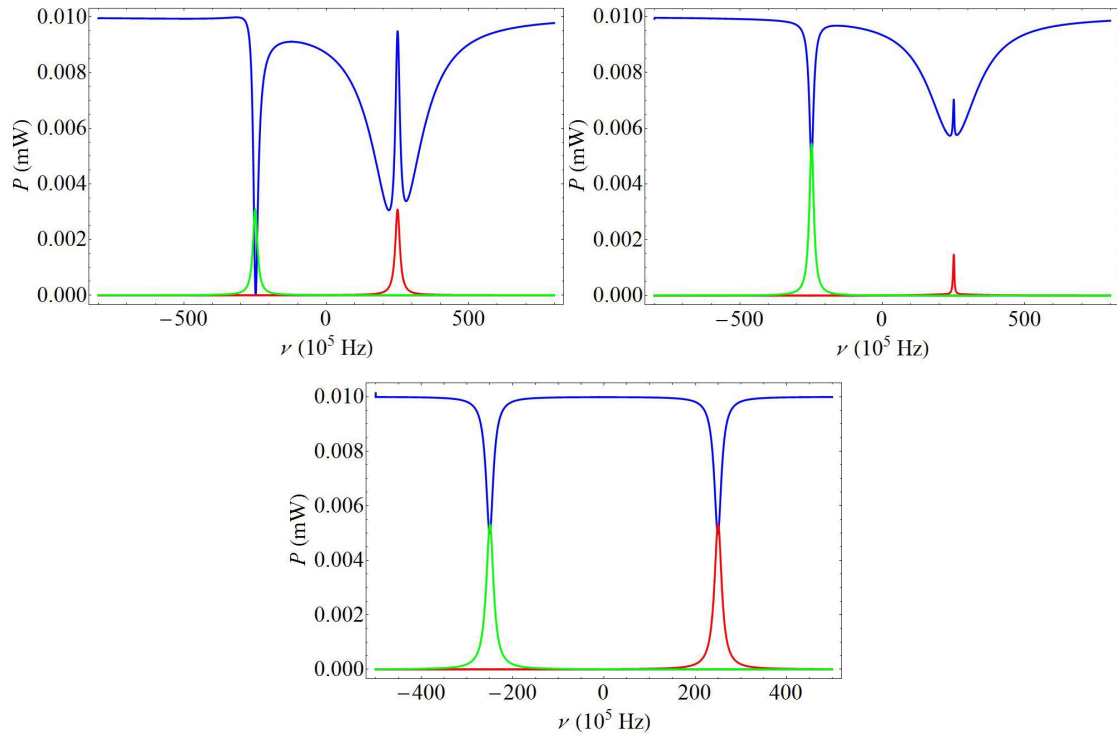


Figure 1. Throughput spectra (dips) and intracavity powers (peaks). Upper left: CPIT. Upper right: FMIT. Lower: single WGM.

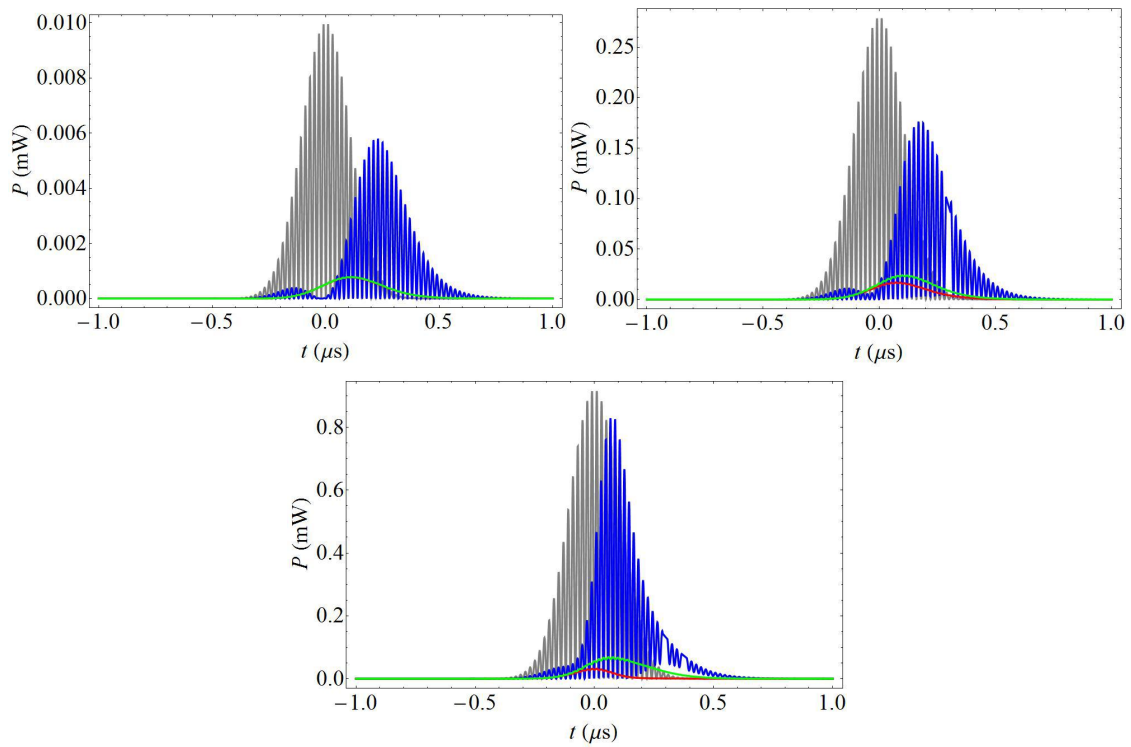


Figure 2. Pulse delay. Modulated input pulse in gray, delayed throughput pulse in blue; delay decreases with increasing input power, as mode 2 intracavity enhancement decreases. (Red curve falls below green curve.)

The resulting pulse (in power) is modulated at the beat frequency, so the appearance of beats due to acoustic excitation just has the effect of slightly distorting the pulse shape. The modulated pulse response shown in Fig. 2 is for the single-overcoupled-WGM case. At low input power (10 μW), there is a delay of about 250 ns. As the input power increases to 280 μW , and then 930 μW , the delay decreases. The decrease in delay occurs as the relative intracavity power in mode 2 decreases, as seen in Fig. 2 as the red curve decreases in amplitude, falling below the green curve (mode 3). This confirms the prediction given in Section 2 that the delay should decrease with decreasing intracavity enhancement.

4. DISCUSSION

The effect of acoustic WGM excitation via forward stimulated Brillouin scattering (SBS) is seen to be detrimental to the production of slow light; it causes the pulse delay to decrease. This is attributed to the decrease in the intracavity power of the higher-frequency optical WGM as SBS transfers some of its power to the lower-frequency optical WGM and the acoustic WGM. Figure 2 illustrated this effect for the single-WGM case, but it is also observed in both induced transparency cases.

Only induced transparency and slow light (pulse delay) have been considered here, but the same model and analysis could also be applied to the case of induced absorption and fast light (pulse advancement).^{3,19} Pulse advancement can also be produced by a single undercoupled WGM, as shown by Eq. (5) for the case $x < 1$. Once again, the prediction will be that the pulse advancement is reduced with a reduction in intracavity enhancement.

An interesting question is whether the slow or fast light could be enhanced by an increase in intracavity power. The model as described above cannot give a conclusive answer, because it involves a single input only. One would need a pump-probe system, with a pump driving mode 2 to produce the SBS, and a probe interacting with mode 3 (or 3 and 1 in the case of CPIT or FMIT), sensitive to the increase in intracavity power of mode 3 above threshold. This is currently under investigation. Experimental study of these effects is also underway, though in preliminary stages at present.

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