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Invited Paper

# Enhanced dissipative sensing in a microresonator with multimode input (theory)

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## ABSTRACT

Optical microresonators, in particular whispering-gallery microresonators, have proven to be especially useful as chemical sensors. In most applications, the sensing modality has been dispersive; an example is the frequency shift of resonator modes in response to a change in the ambient index of refraction. However, it has been shown that the response to dissipative interaction can be even more sensitive than the dispersive response. Dissipative sensing is most often carried out via a change in the mode linewidth owing to absorption in the analyte, but it has been demonstrated that the change in the throughput dip depth of a mode can provide better sensitivity than linewidth change. Dispersive sensing can be enhanced when the input to the microresonator consists of multiple fiber or waveguide modes. Here we show that multimode input can enhance dissipative sensing by an even greater factor. Having multimode input does not affect the linewidth response, but the enhancement factor for the dip-depth response can be quite large. We demonstrate that the multimode-input response relative to single-mode-input response using the same fiber or waveguide can be enhanced by more than three orders of magnitude. Furthermore, this enhancement is independent of the mode linewidth, or quality factor $Q$ of the mode. The enhancement factor can be predicted by making only two measurements of dip depth in the absence of analyte: one with the two input modes in phase with each other, and one with them out of phase.

**Keywords:** microresonator, whispering-gallery modes, dissipative sensing, multimode fiber.

## 1. INTRODUCTION

Optical microresonators have been a topic of much research and application in recent decades. Whispering-gallery mode (WGM) microresonators, in particular, have high quality factors and small mode volumes and hence have been widely used as high-sensitivity sensors. In practical applications, WGM microresonators have been used to monitor changes in pressure, temperature, chemical composition, and refractive index, as well as other quantities.[1]

For chemical sensing, optical WGM sensors detect through the registration of changes in their spectral response due to perturbations in the surrounding environment. The most-used spectral features of a WGM are its resonance frequency and linewidth, used for dispersive and dissipative sensing, respectively. The resonance frequency of a WGM shifts with a change in the refractive index of the surrounding medium, whereas loss mechanisms such as absorption and scattering can affect the linewidth of a WGM. In addition to the change in linewidth, dissipative phenomena can also induce a change in another spectral property, the resonant throughput dip depth, and thus dissipative sensing can be studied in detail by monitoring the change in the dip depth of a WGM.

Previously it was demonstrated that absorption sensing based on relative dip depth change could provide better sensitivity than frequency shift measurements.[2] Experimental evidence[3] was provided by introducing trace gases into the surroundings of a cylindrical fused silica microresonator, and strain-tuning a WGM through a trace gas absorption line. The WGM's effective intrinsic loss gets modified and hence, on resonance with the trace gas absorption line, the dip depth changes by an amount that depends on the external evanescent fraction of the WGM interacting with its surroundings. In addition to the external evanescent fraction, the internal evanescent fraction, which can be much larger than the external, of a hollow microresonator can also be used for sensing purposes.[1,4] A thin-walled hollow bottle resonator (HBR) is ideally suited for use as a WGM-based optical absorption sensor.[4-6] More evidence for dissipative sensing with microresonators being more sensitive than dispersive sensing has recently been provided.[7]

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Recently a novel method for enhancing the sensitivity of evanescent wave WGM based optical absorption sensors was proposed[8] and demonstrated.[9] Typically the WGMs of a microresonator are excited by coupling in tunable laser light using an adiabatically tapered fiber. The enhanced scheme uses a non-adiabatic tapered fiber to couple light into the microresonator and under the right conditions the sensitivity can be enhanced as compared to using an adiabatic taper. The enhancement relies on having multimode input, as illustrated in Fig. 1.

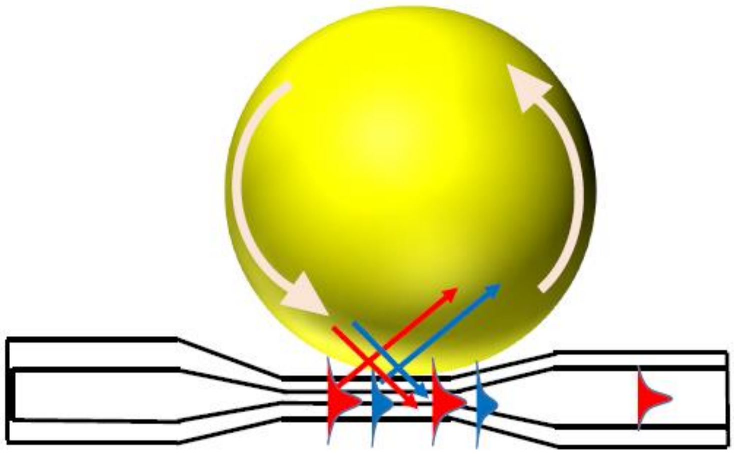

Fig. 1. Coupling of two fiber modes to a single microresonator WGM.

The situation illustrated in Fig. 1 shows a single-mode fiber with an asymmetric bitaper producing a microfiber waist. Because the downtaper is non-adiabatic, two modes are excited on the waist (fundamental in red and higher-order in blue) and couple into the WGM. Outcoupled light goes into both waist modes, but only the fundamental survives passage through the adiabatic uptaper. The resonant throughput dip is monitored for enhanced change due to absorption in an analyte interacting with the WGM's evanescent fraction. The enhancement is relative to the dip depth change that would be observed if only a single waist mode were incident on the microresonator. This technique is generic; the specifics of the waveguide and microresonator do not matter. What is needed is two-mode input, with the two input (waist) modes having different propagation constants; a filter (adiabatic uptaper) to ensure that throughput on only one mode (the fundamental) is detected; and a dissipative loss that changes the net intrinsic loss of the microresonator's mode (such as an absorbing analyte interacting with a WGM's evanescent fraction). The theory presented here thus applies to a large possible range of waveguide-resonator systems. Experimental confirmation of these results is provided in another report in this volume.[10]

## 2. BASIC THEORY

A more detailed sketch of two-mode input coupling to a single microresonator mode is shown in Fig. 2. Two modes are excited and couple to the microresonator mode; it couples back out into the two modes, only one of which gets detected.

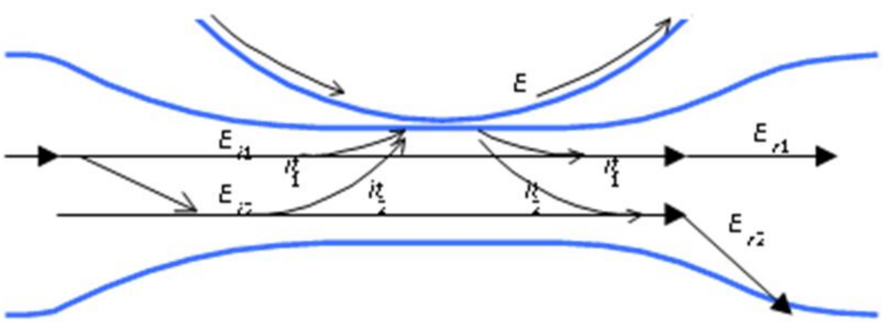


Fig. 2. Two input modes coupling to a single microresonator mode.

The mode amplitudes and coupling coefficients are labeled in Fig. 2. Input modes of amplitudes $E_{ij}$ ($j$ = 1 or 2) couple into the microresonator mode of amplitude $E$ with coupling coefficients $it_j$. Light in the microresonator couples out, with the same coefficients, into the output modes of amplitudes $E_{rj}$, and only the fundamental (mode 1) is detected. The assumption being made here of equality between input and output coupling coefficients is often reasonable,[11] but certainly does not hold in general.[12] The case of inequality will be examined in the next section.

The amplitude of the throughput on mode 1 is given by

$$E_{r1} = \frac{\left(\frac{T_2 + \alpha L - T_1}{2} - i\delta\right)E_{i1} - \sqrt{T_1 T_2}\,E_{i2}e^{i\beta}}{\frac{T_1 + T_2 + \alpha L}{2} - i\delta}, \tag{1}$$

where $T_j = t_j^2$ is the strength of input coupling from and output coupling loss to mode $j$, $\alpha L$ is the round-trip intrinsic loss, $L$ being the microresonator circumference, $\delta$ is the round-trip phase accumulation (modulo $2\pi$), and $\beta$ is the phase of mode 2 relative to mode 1 at the point of coupling into the microresonator. With the definition of $m = E_{i2}/E_{i1}$, the relative throughput amplitude can be written as

$$\rho_{\delta\beta} = \frac{E_{r1}}{E_{i1}} = 1 - \frac{2\left(T_1 + \sqrt{T_1 T_2}\,me^{i\beta}\right)}{T_1 + T_2 + \alpha L - i2\delta}, \tag{2}$$

and the relative throughput power can be written as

$$R_{\delta\beta} = \left|\rho_{\delta\beta}\right|^2 = \left|\frac{\left(\frac{T_2 + \alpha L - T_1}{2} - i\delta\right) - \sqrt{T_1 T_2}\,me^{i\beta}}{\frac{T_1 + T_2 + \alpha L}{2} - i\delta}\right|^2. \tag{3}$$

Equation (3) shows how the throughput spectrum ($\delta$ is proportional to the detuning of the incident light from microresonator resonance) can exhibit different features. For example, for $\beta = 0$ a symmetric dip is normally observed; for $\beta = \pi$ a symmetric peak can appear; and for arbitrary $\beta$ an asymmetric Fano-like lineshape will result. The multimode input can make this Fano lineshape steeper than normal, resulting in enhanced dispersive sensitivity.[13] We show here that even greater enhancement can be produced for the dissipative sensitivity. To that end, note that on resonance, Eq. (3) can be written as

$$R_{0\beta} = \frac{\left(T_1 - T_2 - \alpha L\right)^2 + 4T_1T_2m^2 + 4\left(T_1 - T_2 - \alpha L\right)\sqrt{T_1T_2}\,m\cos\beta}{\left(T_1 + T_2 + \alpha L\right)^2}. \tag{4}$$

This depends on the three quantities $T_1$, $T_2 + \alpha L$, and $T_2 m^2$. Since the total loss is related to the linewidth $\Delta\nu$ of the mode by

$$T_1 + T_2 + \alpha L = \frac{4\pi^2 na\Delta\upsilon}{c}, \tag{5}$$

where $a$ is the microresonator radius and $n$ is the WGM's effective refractive index, the values of the three quantities can be found by measuring the linewidth along with $R_{00}$ and $R_{0\pi}$. The dip depth is given by

$$M = 1 - R_{0\beta} = \frac{4T_1\left(T_2 + \alpha L\right) - 4T_1T_2m^2 - 4\left(T_1 - T_2 - \alpha L\right)\sqrt{T_1T_2}\,m\cos\beta}{\left(T_1 + T_2 + \alpha L\right)^2}. \tag{6}$$

Our dissipative sensing is based on measuring the fractional change in dip depth (with $\beta = 0$) due to additional loss effectively increasing the intrinsic loss by $d\alpha L$; this is expressed as

$$\frac{1}{M}\frac{dM}{d\alpha L} = \frac{T_1\left(T_1 - T_2 - \alpha L\right) + 2T_1T_2m^2 + \left(3T_1 - T_2 - \alpha L\right)\sqrt{T_1T_2}\,m\cos\beta}{\left(T_1 + T_2 + \alpha L\right)\left\{T_1\left(T_2 + \alpha L\right) - T_1T_2m^2 - \left(T_1 - T_2 - \alpha L\right)\sqrt{T_1T_2}\,m\cos\beta\right\}}. \tag{7}$$

If $m = 0$, Eq. (7) gives the fractional change in dip depth for the same additional loss that would be found using the same waveguide or microfiber waist, but with only the fundamental mode incident. The absolute value of the ratio of Eq. (7) with arbitrary $m$ to Eq. (7) with $m = 0$ thus gives the sensitivity enhancement factor:

$$\eta = \left|\frac{\left(T_2 + \alpha L\right)\left[T_1\left(T_1 - T_2 - \alpha L\right) + 2T_1T_2m^2 + \left(3T_1 - T_2 - \alpha L\right)\sqrt{T_1T_2}\,m\cos\beta\right]}{\left(T_1 - T_2 - \alpha L\right)\left\{T_1\left(T_2 + \alpha L\right) - T_1T_2m^2 - \left(T_1 - T_2 - \alpha L\right)\sqrt{T_1T_2}\,m\cos\beta\right\}}\right|. \tag{8}$$

We see that the enhancement depends on the three quantities $T_1$, $T_2 + \alpha L$, and $T_2m^2$, and if they are all nearly equal, the denominator in Eq. (8) becomes small and the enhancement will be large. For example, if $T_2m^2$ is just slightly larger than $T_1$, and $T_2 + \alpha L$ is just slightly larger than $T_2m^2$, it can be seen from Eq. (2) on resonance that the throughput for $\beta = 0$ will show a shallow dip and the throughput for $\beta = \pi$ will have a small peak ($R_{0\pi} > 1$). For $m = 0$, Eq. (2) shows near critical coupling. To illustrate, for any values of $a$ and $n$, if the linewidth is measured to be 15.1 MHz (at 1550 nm wavelength), and $R_{00} = 0.94$ and $R_{0\pi} = 1.04$, we find $\eta = 1298$ and a dip depth of 0.9937 for $m = 0$.

Before analyzing what is meant by this large sensitivity enhancement, consider some of the assumptions made. It is assumed that there are two modes incident, but the microfiber waist has many modes that are above cutoff. Nevertheless, with a proper design of the non-adiabatic downtaper it is possible to excite only the fundamental and one higher-order mode (or at least one cluster of higher-order modes with nearly equal propagation constants).[10] It is further assumed that light in the microresonator couples out into only those two modes. Again, by choosing the ratio of the resonator radius to the fiber waist radius properly, it is possible to achieve this.[11] If there is some outcoupling into even higher-order modes, it will simply appear to be an extra intrinsic loss. The remaining assumption is that the coupling coefficients for waist mode to resonator mode and resonator mode to waist mode are equal. The question is, what results if this assumption is relaxed?

## 3. EXTENDED THEORY

Revisit Fig. 2, and now let the input coupling coefficients be $it_j$, as before, but now call the output coupling coefficients $i\tau_j$. Output coupling is a loss for the resonator mode, so the linewidth gives the total loss again via the relation

$$\tau_1^2 + \tau_2^2 + \alpha L = \frac{4\pi^2 na\Delta\upsilon}{c}. \tag{9}$$

Now measuring $R_{00}$ and $R_{0\pi}$ allows the determination of two other quantities (note that in the case of equal input and output coupling, the three quantities could have been chosen to be $T_1$, $T_1 + T_2 + \alpha L$, and $T_2m^2$), $2\tau_1t_1$ and $\tau_1t_2m$; along with the total loss, these three quantities are all that are needed to determine the enhancement $\eta$. Finding these three quantities from the measured linewidth of 15.1 MHz, $R_{00} = 0.94$, and $R_{0\pi} = 1.04$, we again get $\eta = 1298$. Because the cases of arbitrary $m$ and $m = 0$ involve the same resonator mode and the same input/output coupling coefficients, any effect of input-output coupling difference cancels. Likewise, the total loss does not depend on $m$, so $\eta$ turns out to be independent of the resonator mode linewidth or quality factor $Q$.

This means that the enhancement factor should be expressible in terms of $R_{00}$ and $R_{0\pi}$ only. That expression turns out to be

$$\eta = \left|\frac{1 - \frac{1}{2}\left(\sqrt{R_{00}} - \sqrt{R_{0\pi}}\right)}{\sqrt{R_{00}} - \sqrt{R_{0\pi}}}\right|\frac{2\sqrt{R_{00}}}{1 - \sqrt{R_{00}}} \simeq \left|\frac{2}{R_{00} - R_{0\pi}}\right|\frac{4}{M_{00}}. \tag{10}$$

Once again, entering $R_{00} = 0.94$ and $R_{0\pi} = 1.04$ into Eq. (10), we get $\eta = 1298$. The enhancement factor is seen to be the product of two factors; for the case under consideration, the first factor is approximately 20 and the second factor is approximately 65. The significance of these two factors is discussed in the next section.

## 4. DISCUSSION AND CONCLUSIONS

A dissipative sensing enhancement by three orders of magnitude is remarkable. Nevertheless, one can argue that because $\eta$ is the ratio of response with two modes incident to response with one mode incident, where the one-mode case is very nearly that of critical coupling, a large enhancement is not surprising. As can be seen in earlier work,[2] sensitivity to dissipative processes becomes small in the near-critical-coupling limit. Yet one might want to have the largest intensity possible interacting with the analyte; this requires a large intracavity intensity, which is maximized at critical coupling. Allowing the second incident mode essentially transfers the resulting response to be compared to a much smaller dip depth, effectively amplifying the detection signal.

By considering Eqs. (7) and (8), one can determine how the dissipative sensing signal based on fractional dip depth change compares to the signal based on fractional linewidth change. This gives a way of comparing the two dissipative sensing mechanisms. From the conversion of Eq. (8) to Eq. (10), it can be shown that the second factor of Eq. (10) is the ratio of dip depth signal to linewidth signal; the dip depth method is ~65 times more sensitive.

What about absolute sensitivity? This will depend on the $Q$ and evanescent fraction of the resonator mode. Using a hollow microresonator can provide an internal evanescent fraction ~100 times greater than the external evanescent fraction allowed by a solid resonator. Say the evanescent fraction is fixed; then the greatest sensitivity can be obtained in the single-incident-mode case for strong overcoupling.[2] That sensitivity is greater than the sensitivity for strong undercoupling by a factor that is equal to the ratio of coupling loss to intrinsic loss, which in this limit is approximately $4/M$. This factor is just the second factor in Eq. (10). What this means is that the absolute sensitivity of the two-incident-mode system in the case under consideration, with $Q = 1.28\times10^7$, is the same as that of a strongly overcoupled one-incident-mode system with an *intrinsic* $Q_i = 8.32\times10^8$. An intrinsic quality factor this large is near the limit of what can be achieved in fused silica without taking extraordinary measures; also, keep in mind that a significant evanescent fraction of the microresonator mode is interacting with the analyte (and solvent, if any). For the case considered here, the analyte is a dye in methanol solution, and methanol is a strong (albeit saturable) absorber at this wavelength.

So we can conclude that multimode input permits significant enhancement of microresonator dissipative sensing relative to single-mode input. This enhancement does not depend on the $Q$ of the resonator mode or whether the input and output coupling strengths are equal. Nor does it depend on $n$, $a$, or wavelength. Using the dip-depth-change sensing modality is significantly more sensitive than using the linewidth-change method. Furthermore, the absolute sensitivity achievable using the multimode/dip depth technique is comparable to the very best that can be found using a single-mode method, but with a much more easily produced value of $Q$.

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